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ON THE VISCO-ELASTIC PROPERTIES OF ICE AND WOOD

by

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Abstract

The experiments of this investigation reveal that it is possible to describe the visco-elastic behaviour of ice in a uniaxial state of compression by the same nonlinear rheological model which earlier proved to be suitable for wood. This is the case at least as far as the compressive stress is independent of time and the time period is not very long. The buckling tests as well as certain other tests prove that when the stress changes with time the compression corresponding to each value of stress is dependable on stress history *i.e.* on the law according to which stress changes with time. The extent to which the presented model is then able to describe the rheological behaviour of ice remains beyond the limits of this publication.

1. *Introduction*

The visco-elastic rheological properties of ice have been investigated over a period of a couple of decades (see *e.g.* Tabata [4]). In these investigations one has attempted to describe this phenomenon as with a too complete and thus with a very multi-parametric model. Consequently it is not easy to apply the investigation results into practical use.

Ylinen [7] has presented a nonlinear visco-elastic rheological model for wood which is simply but, however, corresponds well to the real conditions in technical use. On the basis of what is known about the rheological properties of ice there is reason to conclude that the same visco-elastic model can also be adapted to ice.

The investigation capacity of the authors was very restricted owing to a lack of investigation equipment and economical resources. Thus it was necessary to restrict the experiments to the uniaxial state of stress of ice frozen from tap water.

2. *The non-linear visco-elastic rheological model of Ylinen*

Fig. 1 presents a general nonlinear visco-elastic rheological model. Spring 1 is nonlinear while spring 2 is linear and its spring constant is c_2. The dashpot D is a linear viscous damper and η its viscousfriction factor. The following equations are valid:

$$\delta = \varepsilon l = \delta_1 = \delta_2 + \delta_D \, ,$$
$$F = F_1 + F_2$$

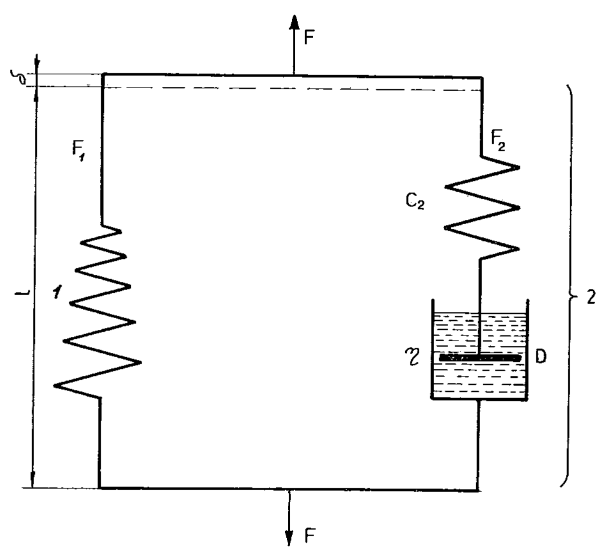

Fig. 1. A nonlinear visco-elastic rheological model.

where (1)

$$F_2 = c_2\delta_2 = \eta\dot{\delta}_D$$

and

$$F_1 = Af_1(\varepsilon)$$

where $A = F/\sigma$ and $f_1(\varepsilon)$ is the ordinate of the stress-strain diagram when the deformation takes place extremely slowly. Further T is the relaxation time

$$T = \frac{l\eta}{AE_2} \tag{2}$$

where $AE_2 = F_2/\varepsilon_2$ with $\varepsilon_2 = \delta_2/l$.

The fundamental differential equation of such a visco-elastic model is

$$\dot{\sigma} + \frac{1}{T}\,\sigma = (E_2 + f_1'(\varepsilon))\,\dot{\varepsilon} + \frac{1}{T}f_1(\varepsilon) \tag{3}$$

where $f_1'(\varepsilon)$ is the derivative of $f_1(\varepsilon)$ while the dot represents the differentiation with respect to time.

Consider that a bar is suddenly loaded by a compressive stress which is held unchanged. Then $\dot{\sigma} = 0$ and ε satisfies the differential equation

$$T(E_2 + f_1'(\varepsilon))\,\dot{\varepsilon} + f_1(\varepsilon) = \sigma\,. \tag{4}$$

The curves in Fig. 2 contain an approximate solution to the above differential equation by YLINEN. The function $f_1(\varepsilon)$ is replaced by the binomial $E_1\varepsilon - a\varepsilon^3$ where E_1 is the modulus of elasticity in an extremely slow deformation and a is a parameter. The curves correspond to the initial condition $t = 0$, $\sigma = E_0\varepsilon - a\varepsilon^3$ where $E_0 = E_1 + E_2$ is apparently the same as the Young's modulus of the material in question. The curves in Fig. 2 were verified by YLINEN with the experiments as follows. A wooden bar was suddenly loaded with a compressive stress *e.g.* $\sigma = 200$ kp/cm^2 and the corresponding ε was recorded immediately. Consequently this numerical value of ε corresponds to the time value $t = 0$. Then σ was held unchanged and ε was recorded at points of time $t_1 = 1$ d, $t_2 = 2$ d, $t_3 = 4$ d etc. The same was repeated with values $\sigma = 300$, 400, 500 kp/cm^2 etc. The thus obtained points took their places on the curves in Fig. 2 with a quite satisfactory accuracy.

3. *The compression tests performed by the authors with ice*

For the compression tests performed by the authors in the refrigerating room of The State Institute of Technical Research the test bars were

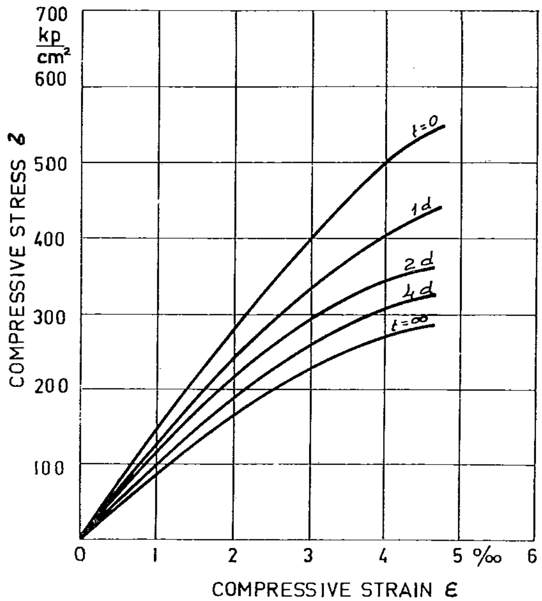

Fig. 2. The compression-test diagrams for wood with the loading time as a curve parameter by YLINEN.

parallelepipeds with a cross-sectional area of ca. 4×4 cm^2 and with a length of ca. 15 cm. The tests were performed in a temperature of ca. -14°C with a device shown in Fig. 3 and owned by the Institute of Marine Research. The compressive strain was measured with two dial gauges shown in Fig. 3. The same hand-driven machine was earlier used in Finnish ice investigations (see *e.g.* SALA [3]). The test plan was the same as the one of YLINEN performed with wood and explained above. The curves which include the test results of the authors for ice are shown in Fig. 4 and they agree, at least qualitatively, with the theory very well.

Unfortunately the possibilities of the authors to perform vast series of experiments were lacking so they are not able to make extensive quantitative conclusions here. In every case it is apparent that the same nonlinear visco-elastic model (Fig. 1) which according to the investigations of YLINEN adapts itself well to the explanation of the rheological behaviour of wood is applicable to ice as well. This perception particularly also contain the fact that all the curves in Fig. 2 and Fig. 4 can obviously be approximated by the same mathematical expression. The parametres in this »deformation law» naturally receive different values for each curve.

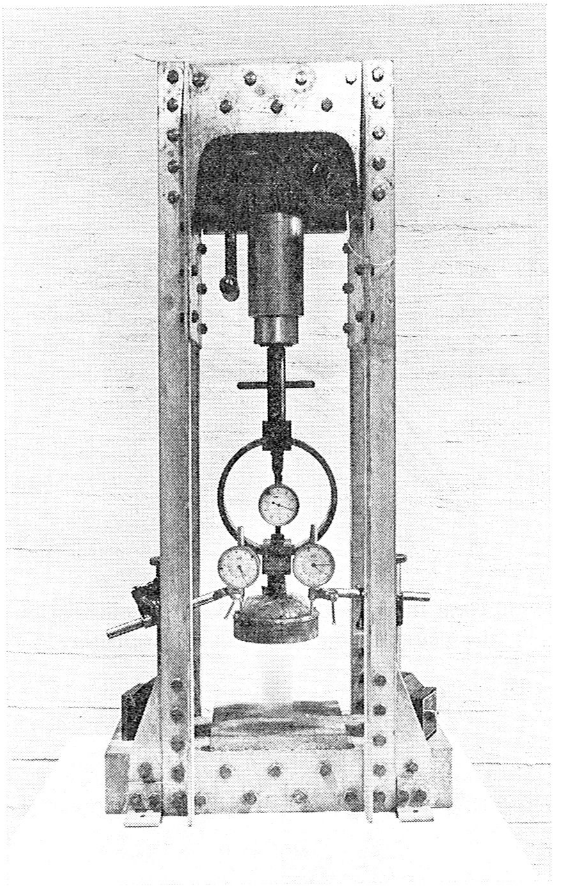

Fig. 3. A hand-driven compression-test machine.

4. *The deformation law of Ylinen and the buckling of a bar*

As it is well known the buckling stress of a bar in the inelastic range is according to Engesser obtained from Euler's formula

$$\sigma_c = \mu \frac{\pi^2 E}{\lambda^2} \tag{5}$$

by replacing the modulus of elasticity E by the so-called tangent modulus *i.e.* by the slope of the compressive stress-strain diagram $E_t = d\sigma/d\varepsilon$ at the point σ_c so that

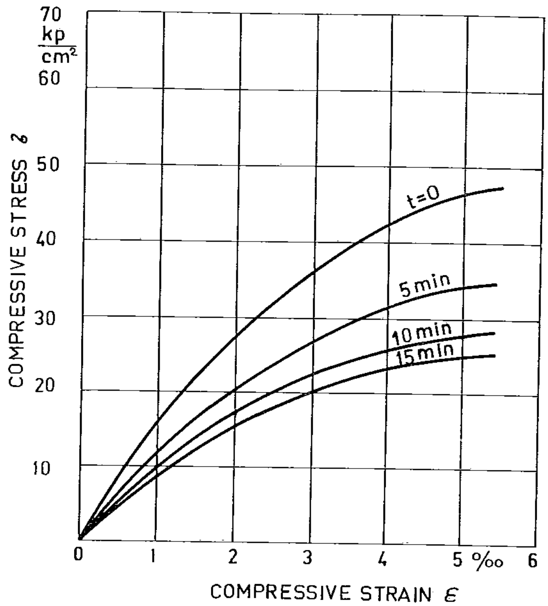

Fig. 4. The equi-time lines of the compressive deformation of ice with the compression stress as a parameter.

$$\sigma_c = \mu \frac{\pi^2 E_t}{\lambda^2} \,. \tag{6}$$

For the tangent modulus Ylinen [5], [6] has suggested an expression

$$E_t = E \frac{\sigma_y - \sigma}{\sigma_y - c\sigma} \,. \tag{7}$$

Here E, σ_y and c are parametres whose values are separately determined for each material so that the above expression represents the slope of the compressive stress-strain diagram of the material in question sufficiently accurately over the whole range $0 < \sigma < \sigma_y$. It is obvious that the parameter σ_y in expression (7) can be identified with the yield point or the ultimate strength, respectively, and the parameter E with the modulus of elasticity of the material in question. Parameter c is to be determined so that the part of the stress-strain curve that falls between the proportional limit and the yield point is represented as accurately as possible. Naturally one can also determine all the three parametres by the compressive stress-strain diagram at the same time (see *e.g.* Neuber [1] and Sala [2]).

When in expression (7) σ is replaced by σ_c and the expression is put in equation (6) into the place of E_t the equation obtained gives

$$\sigma_c = \frac{\pi^2 E + \sigma_y \lambda^2 - \sqrt{(\pi^2 E + \sigma_y \lambda^2)^2 - 4\pi^2 c E \sigma_y \lambda^2}}{2c\lambda^2} \tag{8}$$

provided that the bar has hinged ends so that $\mu = 1$.

YLINEN has for wood determined the values of the parametres in expression (7) separately for each curve in Fig. 2. When these values are put into equation (8) one gets for wood the buckling curves of Fig. 5. In the same way the authors have with the aid of the curves in Fig. 4 calculated the buckling curves for ice in Fig. 6. According to these graphs it seems that the buckling stress considerably decreases as the loading time increases. The publication of YLINEN in question makes no mention of experiments in order to verify the graphs in Fig. 5.

5. *The buckling tests of the authors performed with ice bars*

Above it was revealed that with ice bars curves (Fig. 4) were obtained as test results which as to form are of the same type as those for wood (Fig. 2). Therefore the buckling curves in Fig. 5 and in Fig. 6 are of

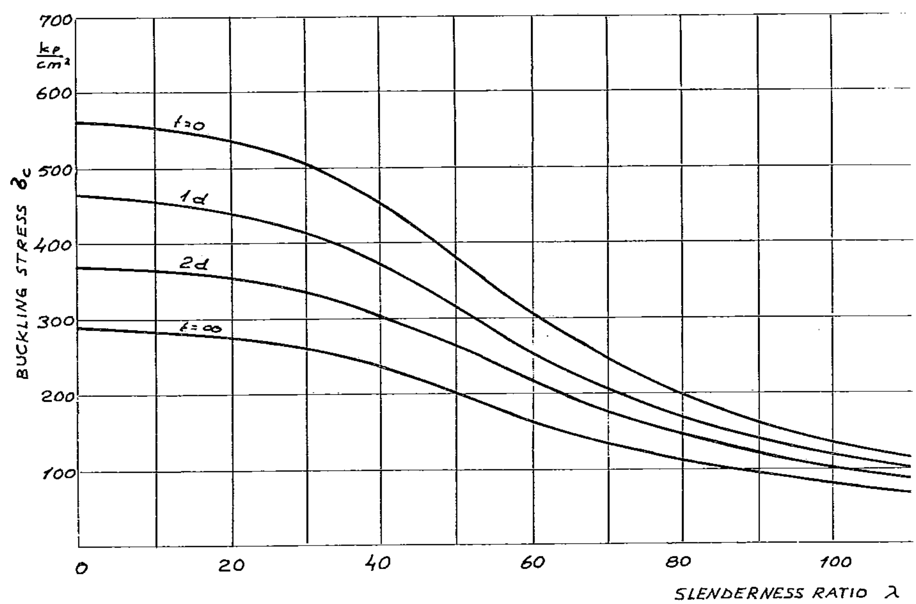

Fig. 5. The effect of the loading time upon the buckling stress of wood by YLINEN.

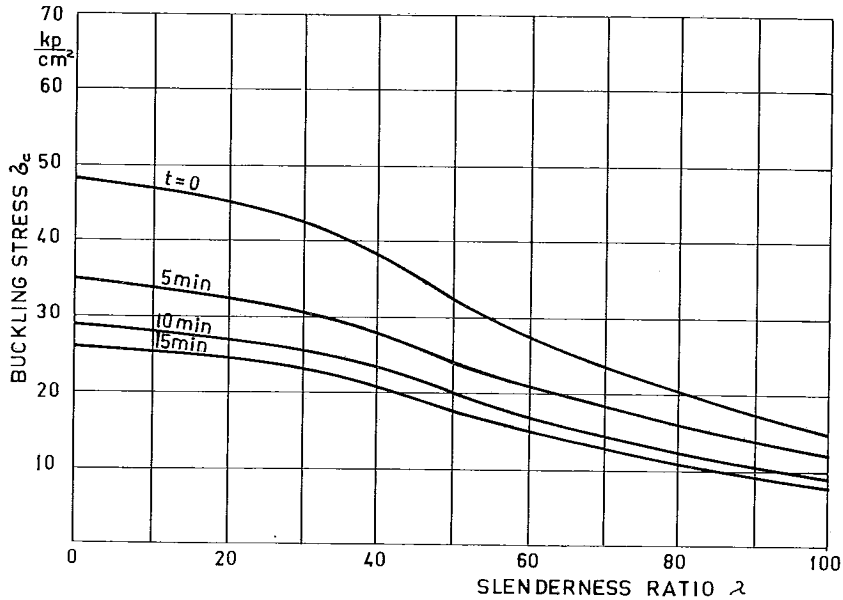

Fig. 6. The »buckling»-curve for ice with the time as a curve parameter.

the same type for both materials. The only essential difference lies again in the scale of σ-axis and in the fact that the visco-elastic deformation of ice takes place considerably quicker than that of wood. The numerical values of the parametres in (7) for ice are determined with the aid of the graphs in Fig. 4 and presented in Table 1.

Table 1. Values of the parametres in (7) for ice

t min.	E 10^3 kp/cm²	δ_y kp/cm²	c
0	18	48	0.4
5	14	35	0.4
10	12	29	0.4
15	11	26	0.4

On the basis of the curves in Fig. 5 it seems that, for example, a wooden bar whose slenderness ratio is $\lambda = 40$ would buckle ca. after half a day provided that it is loaded by a constant compressive stress of $\sigma = 400$ kp/cm² or according to Fig. 6 an ice bar with $\lambda = 40$ and $\sigma = 30$ kp/cm² ca. after 4 min., respectively. The authors performed tests of this kind with bars of ice and with different values of λ and σ

but not a single rheological *i.e.* time-dependent buckling as described above was observed while the buckling stresses in a proper sense *i.e.* the tests results corresponding to the time value $t = 0$ in Fig. 6 agreed very good with the theory. For these buckling tests the bars had a cross-sectional area of ca. 2×4 cm^2 and a length of ca. 20—35 cm. The buckling load were measured with a strain-gauge dynamometer (Fig. 7).

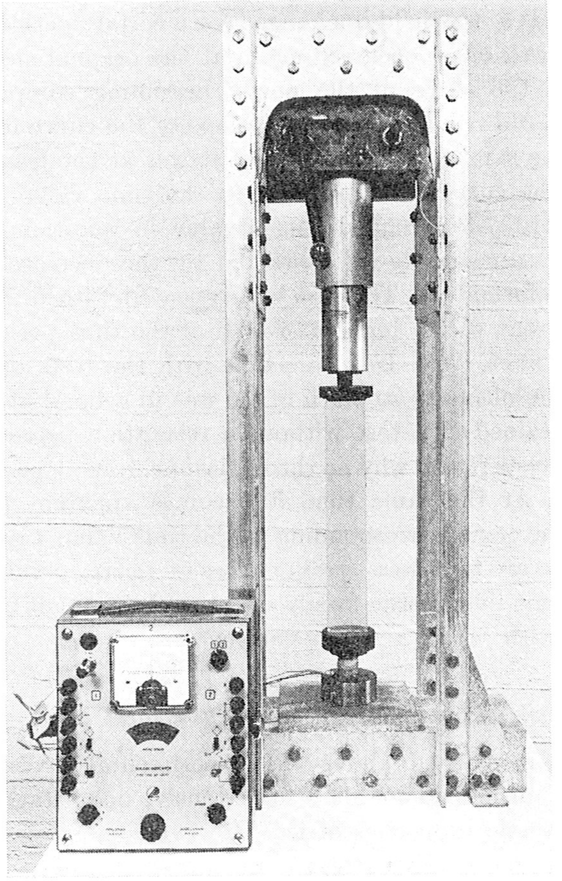

Fig. 7. A device for the buckling tests including the same compressiontest machine as in Fig. 3 with a strain-gauge dynamometer.

6. *The tangent modulus corresponding to the increase in compressive stress which appears suddenly after a visco-elastic deformation*

In order to solve the above discrepancy the authors performed with ice bars the following tests. By these tests they attempted to elucidate whether the slope of each curve in Fig. 4 agrees with the tangent modulus E_t in Eq. (6) which presupposes a rapid deformation that takes place in the buckling. The curve which corresponds to the time value $t = 0$ is the same as the compressive stress-strain diagram of the material in question which is obtained by allowing σ to grow at a set constant speed *e.g.* at 0.2 kp/cm^2 in a second. In the sed tests of the authors a procedure which aimed at such a compressive stress-strain diagram was interrupted and σ was held unchanged for a certain period of time after which the growth of σ occurred again at the original speed. Then one observed that the slope of the newly beginning compressive stress-strain diagram did not agree with the slope of the curve in Fig. 4 corresponding to the same value of time but it was at the least the same as the slope of the curve corresponding to the time value $t = 0$ at the point whose ordinate equalled the σ-value in question, as it is seen in Fig. 8. The same result was obtained by the decrease of σ after a visco-elastic deformation. The test was repeated with different ice bars and with different values both of σ and of the time period. The result was the same each time. In connection with the tests one found that the value of the ultimate strength of ice was in general at least as great as the one obtained in a test without a relaxation period (see Fig. 8).

The above tests reveal why no rheological *i.e.* time-dependent buckling was observed. At the same time it becomes apparent that with the exception of the curve corresponding to the time value $t = 0$ the curves in Fig. 4 for ice are not stress-strain curves of a certain value of t. They are the equi-time lines of the family of integral curves of the differential equation (4) with σ as a curve parameter.

7. *Some conclusions on the visco-elastic rheological properties of ice*

As it was mentioned above the experimental investigations performed by the authors do not allow for extensive quantitative conclusions on the visco-elastic properties of ice. However, one may establish that as long as the compressive stress is independent of time the model in Fig. 1 may suffice to describe the visco-elastic properties of the saltless homogeneous ice at a temperature between -10 and -15°C and in

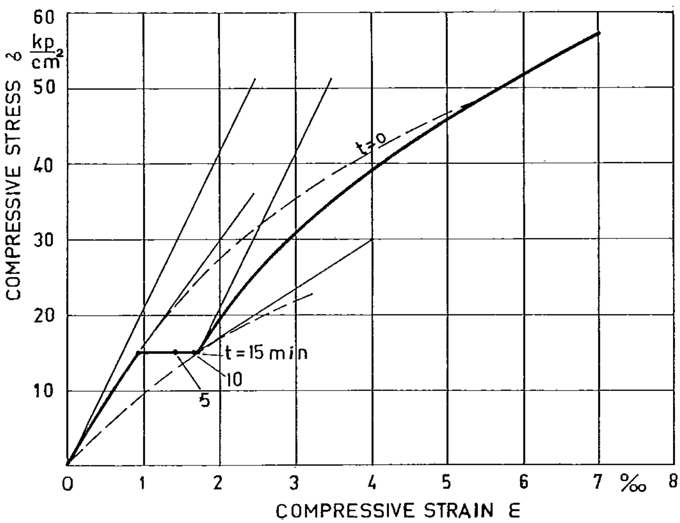

Fig. 8. The compression-test diagram of an ice bar with an interruption in the growth of the compression stress for a time period of 15 min.

time periods which are not very long. On the other hand the buckling tests performed on the ice as well as the tests performed for the determination of the tangent modulus after a relaxation caused by a constant σ clearly show that the value of ε corresponding to a certain value of σ essentially depends on stress history *i.e.* on the law according to which stress changes with time. The extent to which the visco-elastic model shown in Fig. 1 is then able to describe the rheological properties of ice remains beyond the limits of this publication.

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For the experimental part of the present publication OLKKONEN is responsible while SALA is answerable for the theory.

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