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ON ERRORS OF THE HULL PLATE THERMOMETER

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Abstract

Errors of a relatively new method, measurement of temperatures of surface near waters through the hull plate of a vessel, are considered. The crucial factors in this method are the thickness of the hull plate of the vessel and the dimensions of the insulator needed. The obtainable accuracy of the method is comparable to or even better than that of a common mercury thermometer in direct use.

1. *Errors by different temperature measurements*

The common mercury thermometer is a relatively slow instrument. This is because the heat, influencing on the bulb of the thermometer, must flow through the glass wall of low heat conductivity to alter the temperature of the mercury with a considerable heat capasity inside the thermometer. When suddenly moved from a temperature ϑ_0 to $\vartheta_1 = \vartheta_0 + \Delta\vartheta$, the displayed temperature of the thermometer is approximately

$$\vartheta = \vartheta_1 - \Delta\vartheta \exp(-at),$$

where t is the time elapsed counted from the moment of transfer onwards, and a is a constant. To diminish the error $|\vartheta - \vartheta_1|$ to be below the reading accuracy, a certain waiting time, often several minutes, is needed. Mercury thermometers of various constructions are used, however, for different purposes because of their relative simplicity and general accuracy.

The expansive property of liquids is used also in some other devices for measuring temperature. Their properties do not differ very much from those of the mercury thermometer.

The thermocouple is an almost ideal temperature sensor because of its speed. Practically, the accuracy depends only on the instrument used for display, but faulty electrical insulations and electric leads of poor quality may cause trouble. Resistances sensitive to temperature changes and thermistors can also be used as sensors. The former are relatively fast devices. The accuracy of them depends primarily on the displaying instruments. The thermistor is a fast sensor comparable to the thermocouple in its speed and accuracy.

The measurement of surface temperature is often performed by collecting a water sample with a bucket from a moving vessel. This is very difficult in many cases and results may still be unreliable because of many small interfering error sources. But other methods may be used. A suitable device is the hull plate thermometer, which measures the temperature of the inner surface of the hull plating instead of the temperature of the water outside. The sensor and its surroundings are covered by a good insulating material to prevent the heat flow from the surrounding air. In the following, influences of different factors are considered, assuming that a fast sensor is used as measuring device.

2. *Influence of the thickness of the hull plate*

To estimate the error due to the slow penetration of heat through the hull plate, following simple model is formed. Assume, the plate and water outside have a uniform temperature ϑ_0. The insulation at the inner surface, $x = 0$, assumed to be a plane in this paper, prevents the heat flow through this surface. Then the water temperature at the outer surface, $x = d$, and outside is let to jump to another uniform value $\vartheta_1 = \vartheta_0 + \Delta_1\vartheta$. Then the heat begins to flow inside the plate so that

$$\frac{\partial \vartheta(x,t)}{\partial t} = \frac{\lambda}{c\varrho} \frac{\partial^2 \vartheta(x,t)}{\partial x^2},$$

$$\vartheta(x,0) = \vartheta(x,\infty) = \vartheta_1, \ \vartheta(d,t) = \vartheta_1, \ \frac{\partial \vartheta(0,t)}{\partial x} = 0.$$

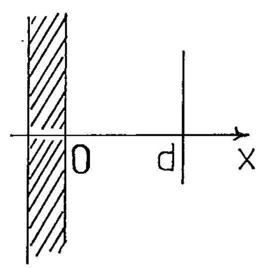

Fig. 1. Insulator, hull plate and water in a one dimensional coordinate system.

Here $\vartheta(x, t)$ = temperature, λ = heat conductivity, c = specific heat, ϱ = density of the plate.
The solution reads

$$\vartheta(x, t) = \vartheta_1 - \frac{4\,\Delta_1\vartheta}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \cos\left[(2n+1)\frac{\pi x}{2d}\right] \exp\left[-(2n+1)^2 \frac{\pi^2 \lambda t}{4 c\varrho d^2}\right].$$

The deficit at the inner surface is thus

$$\delta_1\vartheta = \vartheta_1 - \vartheta(0, t) = \frac{4\,\Delta_1\vartheta}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \exp\left[-(2n+1)^2 \frac{\pi^2 \tau}{4}\right]$$

with

$$\tau = \frac{\lambda t}{c\varrho d^2}.$$

The ratio $\delta_1\vartheta/\Delta_1\vartheta$ is plotted in Fig. 2 as a function of τ and a nomogram for $\delta_1\vartheta\,/\,\Delta_1\vartheta$, t, s is drawn assuming the plate material to be iron with $\lambda = 0.837\ \mathrm{JK^{-1}\,s^{-1}}$, $c = 0.465\ \mathrm{JK^{-1}\,kg^{-1}}$, $\varrho = 7860\ \mathrm{kgm^{-3}}$, see Fig. 3

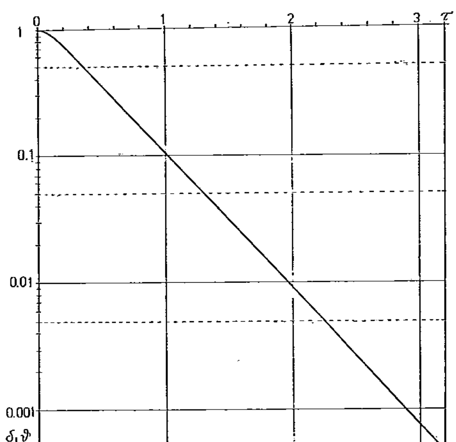

Fig. 2. The ratio $\delta_1\vartheta/\Delta_1\vartheta$ plotted as a function of $\tau = \lambda t/(c\varrho d^2)$.

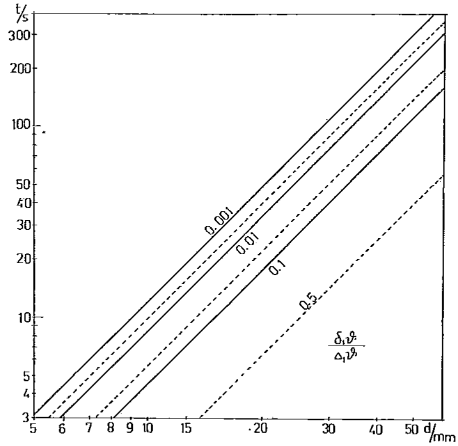

Fig. 3. A nomogram giving the relative accuracy $\delta_1\vartheta/\Delta_1\vartheta$ as a function of the plate thickness d and time t elapsed from the sudden change of the outer temperature assuming the hull plate to be iron.

3. *Influence of the finite dimensions of the insulator*

To simplify the estimation, it is assumed that the insulator is infinite in the y-direction and that it has a finite height h in the z-direction. The temperature sensor be situated at $x = z = 0$ and the insulator at $x = 0$, $-h/2 \leq z \leq h/2$. The outer surface of the hull plate be at $x = d$. The temperature outside the hull, $x \geq d$, be uniformly ϑ_1 and at the cross sections $z = \pm h/2$ uniformly $\vartheta_2 = \vartheta_1 + \Delta_2\vartheta$, a temperature being somewhere between the temperatures inside and oufside the hullplate. The equation for the stationary heat flow is

$$\frac{\partial^2\vartheta(x,z)}{\partial x^2} + \frac{\partial^2\vartheta(x,z)}{\partial z^2} = 0\,,$$

$$\vartheta(x,z) = \vartheta(x,-z)\,,\quad \vartheta\left(x,\frac{h}{2}\right) = \vartheta_2\,,$$

$$\vartheta(d,z) = \vartheta_1\,,\quad \frac{\partial\vartheta(0,z)}{\partial x} = 0.$$

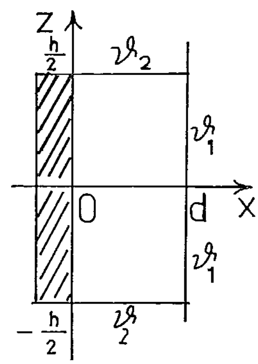

Fig. 4. An insulator of finite height, the hull plate and water in a coordinate system. Also some temperatures are indicated.

The solution is

$$\vartheta(x, z) = \vartheta_1 + \frac{4\,\Delta_2\vartheta}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \cos\left[(2n+1)\frac{\pi x}{2d}\right]$$

$$\frac{\exp\left[(2n+1)\frac{\pi z}{2d}\right] + \exp\left[-(2n+1)\frac{\pi z}{2d}\right]}{\exp\left[(2n+1)\frac{\pi h}{4d}\right] + \exp\left[-(2n+1)\frac{\pi h}{4d}\right]}.$$

Thus, the excess temperature above ϑ_1 at the sensor, $x = z = 0$, is

$$\delta_2\vartheta = \vartheta(0, 0) - \vartheta_1 = \frac{8\,\Delta_1\vartheta}{\pi} \sum_{n=0}^{\infty}$$

$$\frac{(-1)^n}{(2n+1)\left\{\exp\left[(2n+1)\frac{\pi h}{4d}\right] + \exp\left[-(2n+1)\frac{\pi h}{4d}\right]\right\}}.$$

The quantity $\delta_2\vartheta / \Delta_2\vartheta$ is plotted in Fig. 5 as a function of h / d.

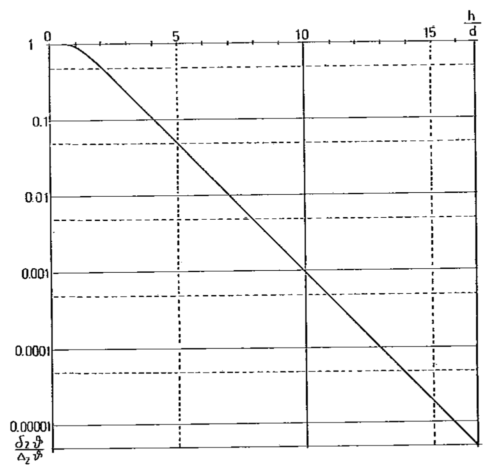

Fig. 5. Relative error $\delta_2\vartheta/\Delta_2\vartheta$ due to the finite height of the insulator as a function of h/d.

4. *Influence of foreign temperatures nearby*

To find an estimate, it is assumed that the insulator of infinite width at $x = 0$ prevents the heat flow through the inner surface. The temperature outside the hull, $x \geq d$ and below the level $z = a$ is $= \vartheta_1$, and at the cross section $z = a$ $\vartheta_3 = \vartheta_1 + \Delta_3\vartheta$. The ensuing stationary heat flow obeys the equation

$$\frac{\partial^2 \vartheta\,(x, z)}{\partial x^2} + \frac{\partial^2 \vartheta\,(x, z)}{\partial z^2} = 0\,,$$

$$\vartheta\,(x, -\infty) = \vartheta_1,\ \vartheta\,(d, \lambda) = \vartheta_1,\ z < a,$$

$$\vartheta\,(x, a) = \vartheta_2,\ \frac{\partial\,(0, z)}{\partial x} = 0\,.$$

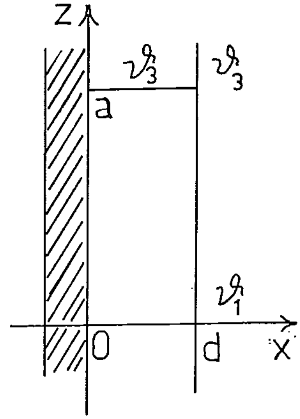

Fig. 6. A foreign heat source in the water above the sensor.

The solution is

$$\vartheta\,(x, z) = \vartheta_1 + \frac{4\,\Delta_3\vartheta}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{2\,n+1}$$

$$\cos\left[(2\,n+1)\,\frac{\pi x}{2\,d}\right] \exp\left[-\,(2\,n+1)\,\frac{\pi\,(a-z)}{2\,d}\right].$$

The excess temperature above ϑ_1 at the sensor is

$$\delta_3\vartheta = \vartheta\,(0\,,\,0) - \vartheta_1 = \frac{4\,\Delta_3\vartheta}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \exp\left[-\,(2\,n+1)\,\frac{\pi\,a}{2\,d}\right].$$

The ratio $\delta_3\vartheta\,/\,\Delta_3\vartheta$ is plotted in Fig. 7 as a function of $a\,/\,d$.

5. *Discussion*

It is seen from the Fig. 3 that after a temperature jump of 10°C, say, the accuracies of 1, 0.1, 0.01°C are attained after 0.68, 1.29, 1.91 min, when a plate thickness of 3 cm is used. These values are better than for an average mercury thermometer. When a smaller boat with a plate thickness of 6 mm is concerned, the time needed for a good temperature reading is only a few seconds.

In the case of the finite insulator, the temperature ϑ_2 is arbitrarily assumed to be uniform, but because of the relatively large values of

h/d, this has practically no influence at all. We may choose for ϑ_2 some values between the inside and outside temperatures ϑ_i and ϑ_1. We have assumed a finite width h to the insulator, but its length be infinite. If we indeed exchange the roles of width and length, we find another estimate for the excess temperature. The two excess temperatures, to be true, are not additive, but a rather realistic value for true excess temperatures can be obtained by adding these different values. This is especially true, when the sum value (sum ratio) is small. Assuming the width and length of the insulator to be equal, the excess temperature will be

$$\delta_2'\vartheta = 2\,\delta_2\vartheta\,,$$

when only small values of $\delta_2\vartheta/\Delta_2\vartheta$ are considered. If the temperature difference $\Delta_2\vartheta$ is taken to be 10°C, then to attain the accuracies of 1.0, 0.1, 0.01°C the width/thickness ratio h/d shall be 5.0, 7.9, 10.9 respectively or for $d = 3$ cm, the widths themselves shall be 15.0, 23.7, 32.7 cm respectively.

Next we consider the effect of a foreign heat source, say the warm water masses above the thermocline. From the Fig. 7 it can be seen that this effect is very small. For example, if we assume the plate thickness to be 3 cm, the distance of an extensive heat source 20 cm from the sensor and the temperature of the heat source 10°C above

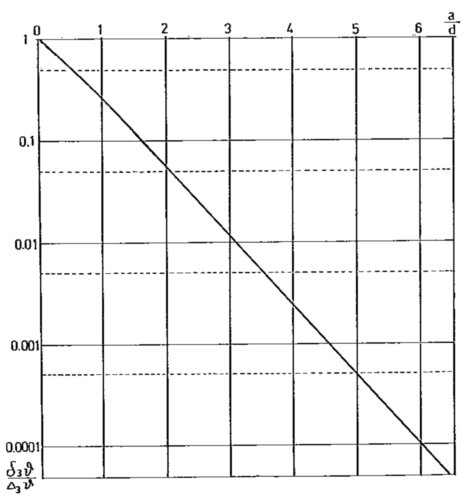

Fig. 7. Relative error $\delta_3\vartheta/\Delta_3\vartheta$ due to the foreign temperature in the water at a height a above the sensor as a function of a/d.

the actual temperature to be measured, then the error due is less than 0.0005°C. There may still be a pitfall. Namely, water is a poor heat conductor compared to iron. Therefore, the water temperature at the plate surface may differ from the water temperatures nearby because of heat flow along the plate, and this is not taken into account in our model.

6. *Conclusions*

On account of the fast action of the temperature outside the hull to the sensor, this method offers a clear advantage over the bucket and mercury thermometer method in determining the water temperatures near the surface. This advantage is still emphasized because there is no worry about the speed of the vessel and the displaying instrument can be accommodated to any place in the vessel, say to the bridge, where the temperature can be read in seconds. A further advantage is that the method is cheap and simple, because, no holes need to be bored into the hull for sensors. Only a well insulated place must be reserved. It is also possible to arrange a continuous recording for measuring temperature variation along the route of the vessel.