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SPECTRAL STUDIES OF TIME SERIES OF THE aa INDEX

by

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Abstract

The maximum entropy method has been applied to spectral studies of two different time series of the aa index of geomagnetic activity: (1) long-time variation of the equinoctial asymmetry in geomagnetic activity and (2) variation of the ratio of geomagnetic activity in the two hemispheres. The former series was found to be quasiperiodic with no dominant period present. In the second series an interesting new peak at the period of four months was found, in addition to the previously known peaks. It remains to be studied whether this peak is an artifact, inherent in the method of determining the aa index.

1. *Introduction*

It is well known that the geomagnetic activity, although highly stochastic in individual appearances, exhibits, when examined statistically, some clear periods, such as the sunspot cycle variation of about eleven years and the semiannual variation with maxima near the equinoxes.

Recently, OKSMAN and KATAJA (1986) have studied the difference of the spring and fall maxima of one measure of geomagnetic activity, the aa index, introduced by MAYAUD (1973). They found that the aa index exhibits unequal spring and fall maxima, the relative dominance of the two equinoxes varying in a quasiperiodic way. They did not perform any detailed spectrum analysis of this variation.

Our first aim in this report is to study the spectrum of the quasiperiodic variation discovered by Oksman and Kataja. We use the maximum entropy method (MEM), well suited for studies of such irregular time series.

As a second topic, we study, using MEM, the spectra of the aa values for the two hemispheres separately and the spectra of their ratio. We check the reliability of the peaks present in these spectra by means of the conventional FFT method.

2. *Method*

The maximum entropy method (MEM) was used to compute the power spectral densities of time series of aa, averaged over one year or one month. MEM is best suited to the treatment of short time series with large noise. That is why the lengths of time series of aa at our disposal are even unnecessarily long. This is especially true in the case of the monthly averages of aa where the length of the time series is 1392 values. In the case of yearly averages there are 116 points, covering the years 1868–1983.

We calculated from the power spectral density plots the time periods corresponding to the various peaks. In MEM the amplitudes of the spectral peaks are not always quite reliable. This problem can be overcome by computing for comparison the power spectral densities also with the FFT method, which gives both the true amplitudes and the phases of the different frequency components.

The MEM power spectral densities were calculated using the MEM algorithms of Burg and Marple. Burg's MEM is based on a constrained least squares estimation procedure using the sum of the forward and backward linear prediction error energies. Marple's MEM is based on an unconstrained least squares estimation procedure.

By the algorithms mentioned above we calculated the coefficients a_k of the formula

$$S(f) = \frac{P_K}{2f_N \left| 1 - \sum_{s=1}^{M} a_K(s)\, exp(-j2\pi f s \Delta t) \right|^2}$$

where

$S(f)$ = power spectral density function,
f = frequency at which the spectral density is estimated,
P_K = total mean-square error for a K-th order autoregressive model of the process,
f_N = Nyquist frequency,
Δt = sampling interval and
M = order of autoregression used to model the process.

a_k:s are computed by numerical methods, and *e.g.* MEM of Burg uses the formulae

$$BA_K(t) = BA_{K-1}(t) - a_{K-1}(K-1)\,BB_{K-1}(t)$$
$$BB_K(t) = BB_{K-1}(t+1) - a_{K-1}(K-1)\,BA_{K-1}(t+1)$$
$$K = 2,3,\ldots,M$$
$$t = 1,2,\ldots,(N-K)$$
$$BA_1(t) = x(t)$$
$$BB_1(t) = x(t+1)$$

3. *Results*

3.1 Equinoctial asymmetry

In Figure 1 the power spectrum of the time series of the ratios of aa_f/aa_{sp} is shown, where aa_f is the average value of aa calculated from the four fall months

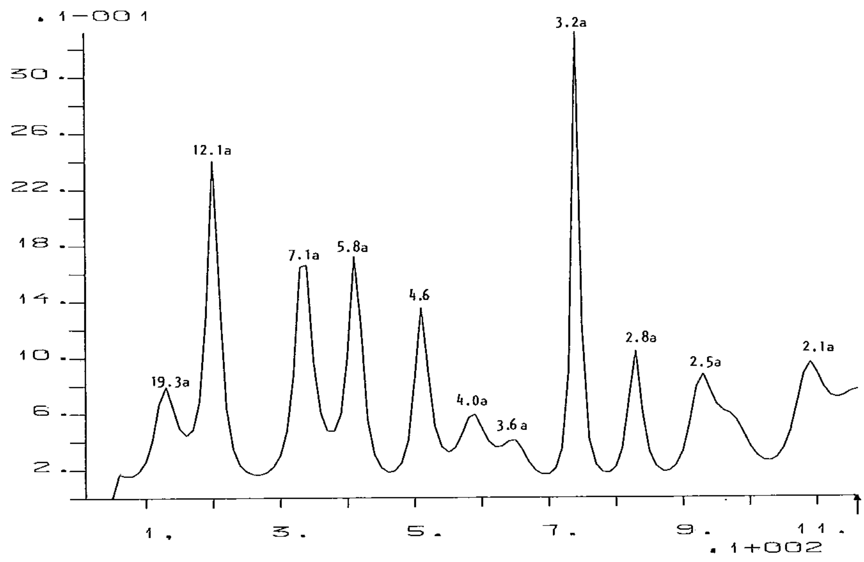


Figure 1. The power spectrum of the time series of the ratio aa_f/aa_{sp} where aa_f is the average value of aa calculated from four fall months (August, September, October and November) and aa_{sp} is the same for the spring months (February, March, April and May). The power spectrum was computed by Marple's MEM (M = 30) and the number of the initial values was 116.

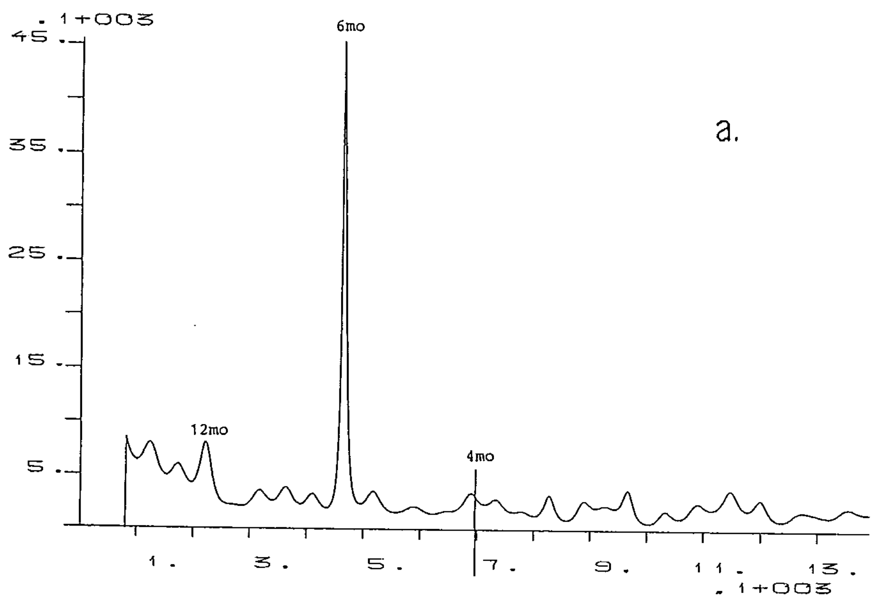
August, September, October and November of each year and aa_{sp} is the same for the spring months February, March, April and May. The spectrum has been calculated from 116 initial values, covering the period 1868–1983, using Marple's MEM with M = 30. The ratio aa_f/aa_{sp} is seen to exhibit several spectral peaks from 2.1 years upwards. It is clearly only quasiperiodic with no dominant period present. The first values of the power spectrum have been omitted because of a large component at zero frequency; thus only time periods shorter than 29 years can be observed.

3.2 Hemispheric asymmetry

In Figure 2 we show the power spectra of the monthly averaged time series of *aa* in the northern (2a) and southern (2b) hemispheres, computed by Marple's MEM. (The results from Burg's MEM are quite similar.)

In the FFT method the number of points used must be a power of two. In our analysis, 1024 last values of the *aa* time series were used (covering the years 1898–1983). (Also 2048 values could be used but then 656 zeros must be added to the end of the time series.) The first values of the power spectrum in Figure 2c have been omitted because of a large component at zero frequency. This means that no periods over 25 months can be observed.

If Figures 2a and 2b are examined closely, it can be seen that the peaks of the *aa*



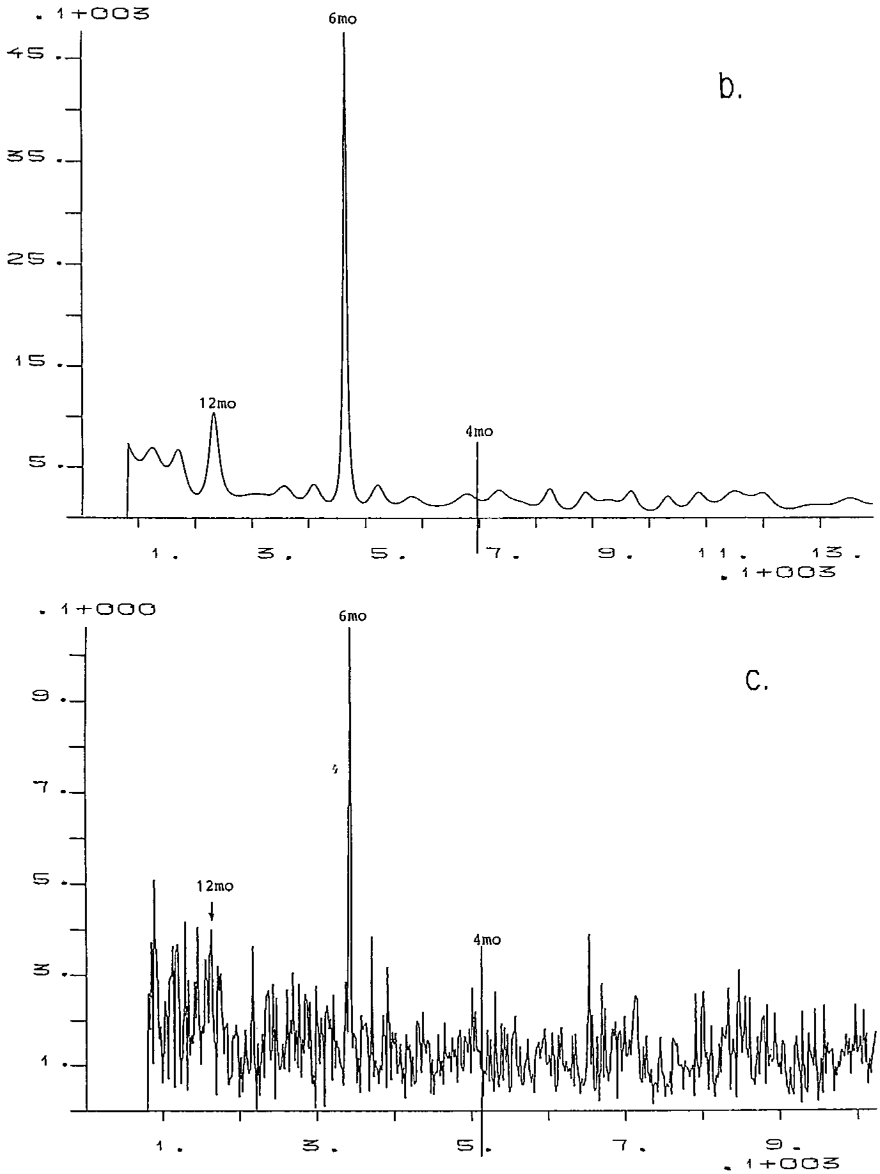


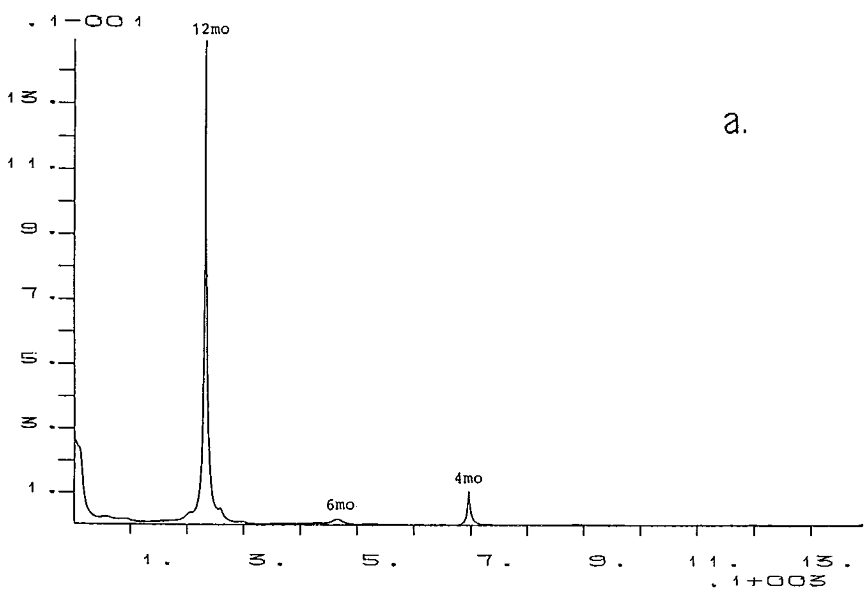
Figure 2. The power spectra of the monthly averaged time series of *aa* calculated by Marple's MEM (2a and 2b, M = 60) and the FFT (2c). 2a and 2c correspond to the northern and 2b the southern hemisphere. The number of initial values is 1392 in 2a and 2b and 1024 in 2c.

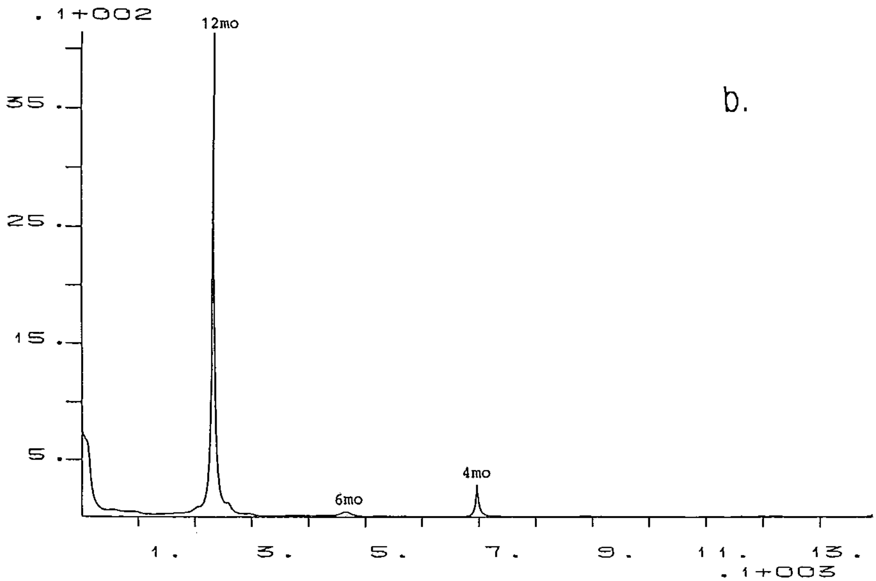
power spectra for the northern and southern hemispheres do not exactly coincide (except for the strong semiannual peaks). The envelope of the FFT spectrum (Figure 2c) exhibits, among others, a weak peak near 4 months. No clear peaks can be discerned in the MEM spectra at this period.

In Figure 3 power spectra of the ratio of *aa* in the northern (aa_n) and southern (aa_s) hemispheres are shown. The spectra were computed from 1392 values of aa_n/aa_s by both Burg's (3a) and Marple's (3b) MEM. The FFT spectrum (3c) was calculated with 1024 last values of aa_n/aa_s.

Especially interesting in the MEM spectra is the very low noise level and the presence of only four clear peaks, corresponding to the periods of infinity (= DC) and 12, 6 and 4 months. These peaks can also be seen in the FFT spectrum.

The more rapid fluctuation of the FFT power spectrum can be explained by the influence of the phase: the FFT phase spectrum (not shown) changes its sign frequently. FFT displays the relative amplitudes of the different frequency components reliably, thus the high peak at four months in 3c enhances the reliability of the corresponding MEM peaks in 3a and 3b.





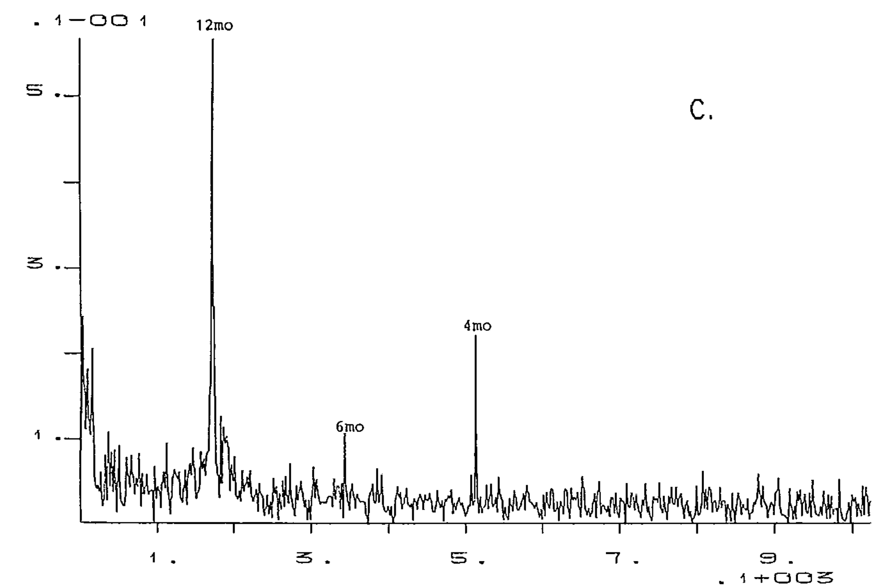


Figure 3. The power spectra of the ratio of *aa* in the northern and southern hemispheres, computed by Burg's MEM (2a, M = 100), Marple's MEM (2b, M = 100) and the FFT (2c). The number of initial values is 1392 in 3a and 3b and 1024 in 3c.

4. *Discussion*

Our result concerning the equinoctial asymmetry of geomagnetic activity confirms the finding of OKSMAN and KATAJA (1986) of this asymmetry being quasiperiodic. The time series of the ratio of the mean *aa* values in the fall and in the spring seems to have several competing periodicities. According to the explanation given by Oksman and Kataja this means that one magnetic hemisphere of the sun dominates over the other for several years in succession, this dominance flipping over in a quasiperiodic way.

The new four month-period in geomagnetic activity found by us is very interesting. This period is almost absent in the time series of *aa* in both hemispheres but is very clearly seen in their ratio. The behaviour of the annual and semiannual peaks is slightly similar: the time series of both hemispheres exhibit strong semiannual peaks and weak annual peaks (Figures 2a and 2b), whereas their ratio displays weak semiannual peaks and strong annual peaks (Figure 3)..The explanation for the latter phenomenon is as follows.

It is known that the semiannual peaks in both hemispheres are caused by a varying coupling between the solar wind and the magnetosphere, this coupling maximising at the equinoxes (RUSSELL and MCPHERRON, 1973). When the ratio of the time series of the two hemispheres is formed, the semiannual variation is almost averaged out but an annual variation (with a maximum in northern summer) emerges, caused by DP2 and local time effects (MAYAUD, 1980).

We have as yet no explanation for the four-month period in aa_n/aa_s. It is possibly caused by the geographical distribution of the stations used for the determination of *aa* values or by some factors related to sun-earth interactions. We have determined the phase of the four-month variation and found that it maximises in the middle of each four-month period of the year (*i.e.* at the end of February, June and October).

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